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# THE FLOW OF ELECTROGASDYNAMIC STREAM OVER A CONDUCTING SPHERE*

O. K. VARENTSOV

The problem of an electrogasdynamic stream flowing over a conducting sphere which simulates an electrostatic probe is solved. Volt-ampere characteristics of the probe are obtained and compared with those derived by approximate theories. The perturbation induced by the probe and associated errors are determined. The numerical analysis is based here on the assumption that the conducting sphere of finite diameter is located between two grid electrodes, one of which simulates the end face of a source of charged particles. Diffusion and inertia of particles are disregarded, and the hydrodynamic velocity field is assumed potential.

An approximate theory of the electrostatic probe was proposed in /1,2/, without allowance for the perturbation by the probe of the electric space charge, on the assumption of the external field homogeneity and zero velocity of the gas stream; diffusion and inertia of charged particles was disregarded. The flow of an electrogasdynamic stream over a cylindrical probe was considered in /3/ on the same assumptions, except that velocity was not zero. The effect of gas compressibility on indications of the probe was approximately evaluated. The presence of a special shape probe which did not disturb the electric streamlines was investigated in /4/. In the experimental investigation /5/ the simplest theory which does not take into account the effects of the electric space charge near the probe and of its polarization was used.

To determine the limits of applicability of approximate theories it is necessary to solve the more general problem of the flow of a stream with electrically charged particles over the probe. That solution is presented below.

## 1. Statement of the problem and the method of solution.

Let us consider a conducting sphere of radius $a^\circ$ positioned between two parallel infinite grid electrodes. We select the cylindrical coordinate system so that the axis $Ox^\circ$ is normal to the electrode planes whose equations are $x^\circ = 0$ and $x^\circ = L$, and passes through the sphere center $x^\circ = x_0^\circ$, $y^\circ = 0$.

We assume that the grids are grounded, $\varphi^\circ(0, y^\circ) = \varphi^\circ(L, y^\circ) = 0$, permeable to gas, that it is possible to nelgect the effect of all thin supply circuits on the distribution of electric and hydrodynamic quantities in the inter-electrode gap, and that the sphere potential $\varphi_s^\circ$ can be controlled by an external voltage source. Parameters of the electrogasdynamic interaction are assumed small, which means that the distribution of gasdynamic parameters in the space surrounding the sphere are determined by conventional gasdynamic equations. It is further assumed that the fluid is incompressible, its viscosity negligible, and that the stream homogeneous at infinity where its velocity $\mathbf{U}_0$ is parallel to the $Ox$-axis. Under these conditions the velocity field in the space surrounding the sphere is potential.

The electrode $x^\circ = 0$ can be considered as the end face of the charged particle source, and is assumed to work so that near the electrode $x^\circ = 0$ the electric space charge density $q^\circ(0, y^\circ) = q_*$ is constant. Charged particles are carried by the stream of fluid from the emitter- to the collector-electrode. Their inertia is assumed negligible (e.g., when the particles are ions) and the Péclet number high, which makes also the diffusion of particles negligible. Ohm's law for a charged component is of the form $\mathbf{j}^\circ = q^\circ(\mathbf{v}^\circ - b\nabla\varphi^\circ)$, where the particle mobility $b$ is assumed constant.

On these assumptions the distribution of potential $\varphi$ and space charge $q$ in the inter-electrode gap is defined in the dimensionless coordinates

$$\varphi = \frac{b}{U_0 L}\varphi^\circ, \quad q = \frac{4\pi b L}{\varepsilon U_0} q^\circ, \quad x = \frac{x_\circ}{L}, \quad y = \frac{y^\circ}{L}, \quad \mathbf{v} = \frac{\mathbf{v}^\circ}{U_0}$$

by the following system of equations and boundary conditions:

$$\Delta\varphi = -q, \ (\mathbf{v} - \nabla\varphi)\nabla q + q^2 = 0 \qquad (1.1)$$

$$x = 0, \ \varphi = 0, \ q = \beta; \quad x = 1, \ \varphi = 0$$

$$(x - x_0)^2 + y^2 = a^2, \quad \varphi = \varphi_s; \ y = \infty, \ \partial\varphi / \partial y = 0 \qquad (1.2)$$

$$\beta = \frac{4\pi b L}{\varepsilon U_0} q_*, \quad \varphi_s = \frac{\varphi_s^\circ b}{U_0 L}, \quad a = \frac{a^\circ}{L}, \quad x_0 = \frac{x_0^\circ}{L} \qquad 1.3)$$

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where $\mathbf{v}^\circ$ is the known velocity distribution for a potential flow over a sphere.

Previously obtained solutions /3/ related to the simplified problem $\Delta\varphi = 0$, $(\mathbf{v} - \nabla\varphi)\nabla q = 0$; $q = q_\infty = \text{const}$, and $\mathbf{E} = \mathbf{E}_\infty = \text{const}$, as $\mathbf{r} \to \infty$. In /2—5/ the effect of $q$ on the potential distribution and the variation of electrical parameters away from the sphere were disregarded, and the electric charge in the region occupied by particle trajectories was assumed uniform.

In the above formulation the problem is defined by four dimensionless parameters (1.3). In the absence of a sphere in the stream the potential $\varphi_0$ and the charge density $q_0$ at point $x_0$ are determined by the solution of the one-dimensional problem of electrogasdynamics /6/

$$\varphi'' = -q, \quad (1 - \varphi')q' = -q^2, \quad \varphi(0) = \varphi(1) = 0, \quad q(0) = \beta$$

An "ideal" probe must, therefore, register the quantities $q_0$ and $\varphi_0$.

System (1.1) was numerically integrated using a rectangular grid which was nonuniform near the sphere. Zeidel's method was used for integrating the Poisson equation, and the method of characteristics for integrating the equation of the charge. The solution of system (1.1) was derived by the process of successive approximations /7/.

In a number of calculations in which the sphere dimension was successively reduced, the computation region was simultaneously decreases. Because of this, region $D^\circ$ was computed for $a = a_1$. In that region the perturbations of one-dimensional electrogasdynamic stream are zero within the accuracy of the numerical method. Since perturbations of an electrogasdynamic stream decrease as the sphere radius is decreased, the solution in region $D^\circ$ is the same, when $a = a_2 < a_1$, (within the same accuracy) as the one-dimensional solution. Hence region $D^\circ$ can be eliminated from the investigation.

The accuracy of the control of full current maintenance over the cross section $x = \text{const}$ was not less than 3%. Moreover, as radius $a$ was reduced, the ratio of current on the grounded sphere to its radius $I_s / a$ approaches $I_{s0} / a = 4\pi q_0 \varphi_0$, the value yielded by Sato's theory, i.e. the calculated limit coincides with the known analytic limit.

2. The flow pattern. Using the simple reasoning of /2,5/ it is pssible to show that, depending of the probe potential $\varphi_s$, different modes are realised when charged particles reach the whole surface of the probe, or a part of it, or altogether fail to reach the probe. In the latter case the probe is surrounded by region $D$ which is free of charged particles and

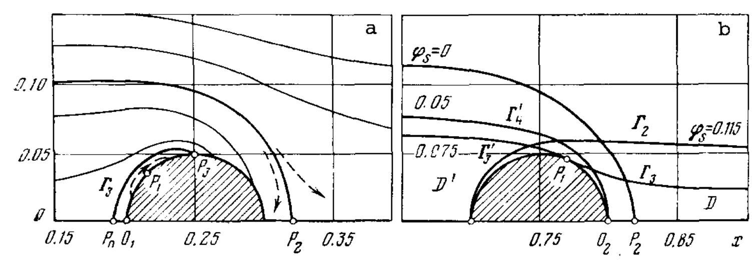


Fig.1

may be either closed or infinitely extended /3/. When $\varphi_s < \varphi_f$ ($\varphi_f$ is the minimum potential—"the floating" potential— at which capture of particles by the probe occurs), region $D$ adjoins only some part of the probe surface and vanishes as the potential $\varphi_s$ decreases within the interval $(\varphi_*, \varphi_f)$. All these modes: mode 1 of closure $(\varphi_s > \varphi_f)$, 2 of the floating potential $(\varphi_s = \varphi_f)$, 3 transition modes $(\varphi_* < \varphi_s < \varphi_f)$, and 4 the mode of complete selection of particles which obtains when $\varphi_s \leqslant \varphi_*$. These modes have been analized in numerical problem (1.1) and (1.2).

As an example, the streamlines around a sphere of radius $a = 0.05$ located at the point at coordinate $x_0 = 0.25$ are shown in Fig.1a. The degree of stream saturation was $\beta = 4$ and the sphere potential $\varphi_s = 0.09$. The zone comprized between curve $\Gamma_3$ and the sphere surface is not reached by charged particles. It is filled by characteristics of the second of Eq. (1.1) which begin at section $O_1P_1$ of the sphere surface and finish on sections $P_1P_3$, where point $P_1$ is determined by the condition $\partial\varphi / \partial n = 0$. The case in which the charge-free region is bounded downstream only by the collector-electrode is illustrated in Fig.1,b. The various positions of boundary $\Gamma_i$ of region $D$ correspond to different potentials on the sphere. In mode 2 $(\varphi_s = \varphi_f = 0.115)$ and boundary $\Gamma_2$ touches the sphere at the forward stagnation point. In mode 3 $(\varphi_s = 0.075)$ and curve $\Gamma_3'$ which is the continuation of boundary $\Gamma_3$ of the upstream charge-free zone without of charge $D$, bounds region $D'$ from which all particles are caught by the sphere. When $\varphi_s = \varphi_* = 0.05$ the charge-free zone degenerates into segment $O_2x$ and curve $\Gamma_4'$ touches the sphere at the rear stagnation point.

Note that potentials $\varphi_{f0}$ and $\varphi_{*0}$ which correspond to the model in /3/ are not the same as the calculated $\varphi_f$ and $\varphi_*$. The presence of an electric field in the stream and the allowance for perturbations of the electric space charge induced by the sphere alters the characteristic values of the potential. In the case shown in Fig.1 b the floating potential is less than $\varphi_{f0} = 0.133$, owing to the decrease upstream of $E_x$. The difference between $\varphi_*$ and $\varphi_{*0} = 0.053$ is due to the effect of the grounded collector and the absence of a charge in region $D$; both factors impede the transition to the total extraction mode.

Let us consider the charge distribution in the interelectrode gap. The determination of $q$ on line $\Gamma$ and on the axis of flow downstream of the sphere requires the determination of variation of $q$ in the neighborhood of points $P_0$ and $P_2$ (Fig.1) at which the particle velocity is zero.

Since points $P_0$ and $P_2$ are similar, it is sufficient to consider only point $P_2$.

We expand $E_y$ and $v_y$ in terms of $y$ in the neighborhood of point $P_2$, taking into account the continuity and boundedness of $\partial E_x / \partial x$ and $q$ at that point. We have

$$E_y = -\alpha_1 y + O(y^2),\ v_y = -\alpha_2 y + O(y^2),\ \alpha_1 = (\partial E_x / \partial x - q)_{P_2} \tag{2.1}$$

where $\alpha_2$ is obtained using formula for the velocity of a potential flow over a sphere.

Let us investigate the characteristics of the second of Eqs. (1.1). The equation of characteristics and the relations along them are of the form

$$\frac{dy}{dx} = \frac{v_y + E_y}{v_x + E_x},\quad \frac{dq}{q^2} = -\frac{dy}{v_y + E_y} \tag{2.2}$$

Integrating (2.2) with allowance for (2.1), we obtain the following law of variation of $q$ along the characteristic in point $P_2$ neighborhood:

$$q = \left(C - \frac{1}{\alpha_1 + \alpha_2} \ln y\right)^{-1},\quad C = \text{const} \tag{2.3}$$

which implies that in that neighborhood $q$ is small, $(q)_{P_2} = 0$ and $\alpha_1 = (\partial E_x / \partial x)_{P_2}$. It is also obvious that $P_2$ is a singular point of the "saddle" type.

As implied by (2.2), parameter $q$ always decreases or is zero on charged component streamline. Thus along the boundaries $\Gamma_1$ and $\Gamma_2$ in the floating potential and closing modes, as well as along the axis downstream of the sphere, $q = 0$ in any mode. Consequently, boundary $\Gamma$ is a discontinuity line of $q$ only in the transition mode, when the stream does not contain points $P_0$ and $P_2$.

Numerical computations had shown that in mode 4 the distribution of $q$ in the region downstream of the sphere is preturbed (as compared with the uniform flow in the absence of a sphere) only in the narrow (of the order of the sphere diameter) zone adjacent to the stream axis. In the region upstream of the sphere charge perturbations occur in a wider zone, reaching their maxima at the stream axis and vanishing as $y \to \infty$.

The radial distribution of $q$ at cross section $x = 0.05; 0.15; 0.25; 0.30; 0.55; 0.95$ (curves 1– 6, respectively) in the stream flowing over a grounded sphere of radius $a = 0.05$ whose center is at point $x_0 = 0.25$; at a stream saturation $\beta = 4$ is shown in Fig.2. Longitudinal distributions of $q$ and $\varphi$ are also shown there for the same governing parameters. The dash lines correspond to undisturbed one-dimensional distributions of $q_0(x)$ and $\varphi_0(x)$.

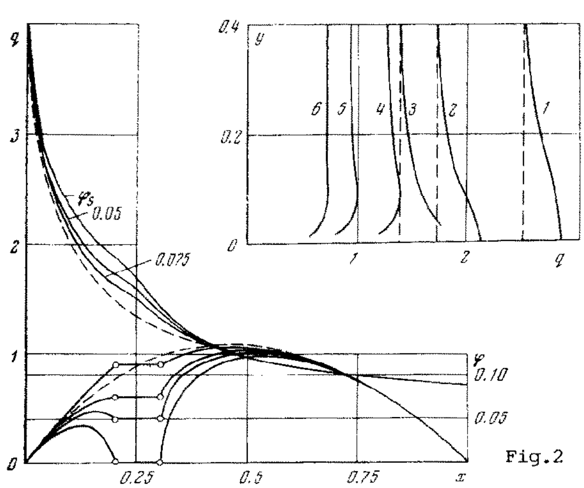


Fig.2

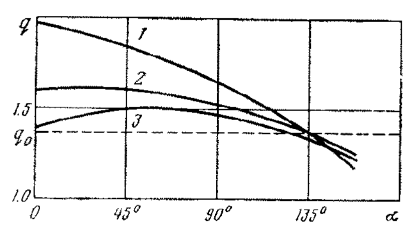


Fig.3

These variations of the space charge in the neighborhood of the sphere produce a nonuniform distribution of $q$ over its surface. In the vicinity of the rear stagnation point the charge $q$ is always lower than its unperturbed value $q_0$ due to the greater length of streamlines reaching that zone, and to the low velocity $\mathbf{v}_i$ charges ($\mathbf{v}_i = \mathbf{v} + \mathbf{E}$) near the point of turn of these lines. As the sphere potential is lowered, velocity $\mathbf{v}_i$ of particles reaching it increases, which in conformity with (2.3) leads to the increase of $q$ and of the nonuniformity of space charge distribution over the sphere surface.

The distribution of $q$ over the sphere surface is shown in Fig.3, where angle $\alpha$ is measured from the forward stagnation point. Curve 1 corresponds to the example shown in Fig.2, and curves 2 and 3 relate to $\varphi_s = 0.05$ and $0.075$, respectively.

The perturbation of charge $q$ essentially depends on the position of the sphere in the interelectrode gap. Computations have shown that a grounded sphere set near the collector at $x = 1$ virtually does not affect the space charge density distribution.

For instance, for $\beta = 4$, $\varphi_s = 0$, $x_0 = 0.75$, and $a = 0.05$ the relative perturbation of the charge does not exceed 2%. The only exception is the narrow zone close to the axis downstream of the sphere, where charge $q$ vanishes. The perturbations of $q$ considerably increase when the sphere is brought closer to the emitter. Thus at $x_0 = 0.50$ and 0.25 and fixed remaining parameters, the relative perturbations of $q$ near the sphere is $\sim 10$ and 24%, respectively.

The increase of the sphere perturbing effect on electrical parameters when it is brought closer to the emitter is due to that near the latter the charged particle velocity $v_x + E_x$ is low owing to the considerable braking field $E_x$ and, consequently, small variations of the electric field lead to considerable perturbations of the density of charged particles.

An increase of the stream saturation by the electric charge is accompanied by the increase of the braking field at the emitter, with the corresponding fall of the particle "starting" velocity to zero /6/. This leads to the increase of perturbation of the space charge density with increasing parameter $\beta$.

When the source operates in the saturation mode $(\beta = \infty)$, it is possible to estimate the increase of the charge space density. We shall make use of the fact that, when $\beta = \infty$, the condition $E_x = -1$ is satisfied at the emitter, which means that the over-all charge density induced on the emitter by the charged sphere and the additional space charge $\Delta q = q(x, y) - q_0(x)$ is zero at any point of the emitter. When calculating the induced charge the sphere was replaced by the point charge $Q_s = -a(\varphi_0 - \varphi_s)$. Moreover, taking into account that in the sphere downstream region the perturbation of $q$ is small, it was assumed that in the interval $(x_0, 1]$ $\Delta q(x, y) \equiv 0$. As the result, we obtained the estimate

$$\Delta q_s = 2a(\varphi_0 - \varphi_s)x_0^{-3} \tag{2.4}$$

where $\Delta q_s$ is some mean value of the additional charge in the sphere neighborhood. This estimate is in a satisfactory agreement with the one obtained by numerical calculations for $\beta \geqslant 4$.

## 3. The volt-ampere characteristics of the spherical probe.

The method of determination of the space charge potential and density in an electrogasdynamic stream is based on the analysis of the probe volt-ampere characteristic. In the calculation of current flowing to the probe it is usual to assume that the diffusion currents are neglibibly small. If the charged particles reach the whole surface $\Sigma$ of the probe, we have

$$I_s = -\int_\Sigma qE_n\,d\sigma = -\langle q\rangle \int_\Sigma E_n\,d\sigma = -\langle q\rangle Q_s \tag{3.1}$$

where $Q_s$ is the sphere charge. In the simplified theory of the probe it is assumed that /2,5/

$$\langle q\rangle = q_0,\quad Q_s = C(\varphi_s - \varphi_0) \tag{3.2}$$

where $q_0$ and $\varphi_0$ are, respectively, the space charge density and potential at a given point of the stream in the absence of a probe, and $C$ is the probe capacitance relative to the electrodes. Note that for an incompressible fluid formula (3.1) also holds in the case when zone $D$, free of space charge and of closed form, adjoins the probe. The boundary of zone $D$ is formed by the $\Sigma_0$ part of the probe on one side and by surface $\Sigma_3$ with the generatrix $\Gamma_3$ (Fig.1) on the other. Then, taking into account the impermeability condition for the charged component on $\Sigma_3$ and for gas on $\Sigma_0$, for the stream of vector $\mathbf{E}$ through $\Sigma_0$ we obtain

$$\int_{\Sigma_0} E_n\,d\sigma = \int_{\Sigma_0} (v_n + E_n)\,d\sigma = \int_{\Sigma_0+\Sigma_3} (v_n + E_n)\,d\sigma = \int_D \operatorname{div}(\mathbf{v} + \mathbf{E})\,d\tau = 0$$

which shows that it is possible to integrate in (3.1) over the whole surface $\Sigma$.

Formulas (3.2) are approximate. The sufficiency of the first of them may be judged by the results of the above numerical calculations. The second is to be compared with the more exact formula /8/ containing the additional term which takes into account the charge "displaced" by the probe

$$Q_s = C(\varphi_s - \varphi_0) + Q_e + q_0V_s,\quad Q_e = \int_\Sigma (\varphi_0 - \varphi)\,c\,d\sigma \tag{3.3}$$

where $\varphi$ is the distribution of potential in the absence of a sphere and $c\,d\sigma$ is the capacitance of an element of the sphere surface. Hence the error of the probe potential determination by the second of formulas (3.2) without allowance for space charge perturbation is

$$\Delta\varphi_0 = (q_0 V_s + Q_e) / C \tag{3.4}$$

With an accuracy to terms of higher order of smallness, $Q_e$ can be represented in the form

$$Q_e = \int_S E_0 (r - r_0) c \, d\sigma$$

which shows that, when the electrodes are grounded, $Q_e$ is always greater than zero, and infinitely increases as the probe is brought closer to them.

Thus the simplified theory generally yields lower values of the potential. The error $\Delta\varphi_0$ of potential determination is given by (3.4); it tends to zero as the probe size approaches zero. Computations have shown that variation of the probe charge due to perturbation of the electric space charge density is small, and in all cases considered here did not exceed 0.5% of the over-all charge of the probe.

The calculated volt-ampere characteristics of a spherical probe of radius $a = 0.05$ in a one-dimesnional stream $\beta = 4$ are shown in Fig.4 with the probe at points $x_0 = 0.75$ and $x_0 = 0.25$ (curves $A$ and $B$, respectively). The curves of $I_{s0} / (4\pi a) = q_0 (\varphi_0 - \varphi_s) = I_{s0}^\circ$ are shown there by dash-dot lines, and the solid lines correspond to the calculated values of $I_s^\circ = I_s / (4\pi a)$, where $I_s$ is the current absorbed by the sphere and $4\pi a$ is the capacitance of an isolated sphere.

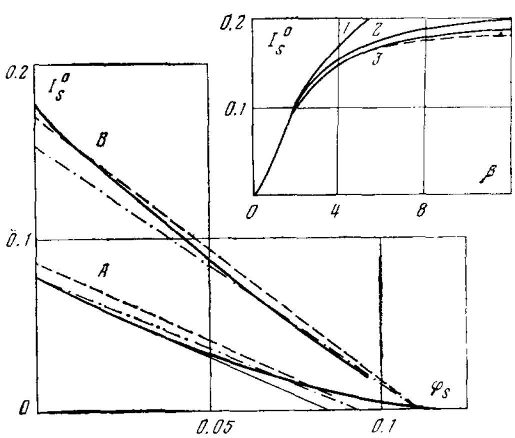


Fig.4

The shift of the characteristic to the left is due to the increase of $Q_e$ as the probe approaches the electrode. Charge perturbations are virtually absent and the increase of the volt-ampere characteristic slope is due exclusively to the increase of the probe capacitance as its distance from the electrode is diminished. The curve of $I = I_{s0}^\circ C_1 / (4\pi a)$, where $C_1$ is the sphere capacitance relative to the electrode is shown by the dash line for comparison.

Thus the approximate theory can be applied in investigations of regions downstream of the point of maximum potential, provided that the probe is at a fair distance from the conducting surfaces. If this condition cannot be satisfied, the measured value of the potential can be corrected with the use of (3.4). The error of determination of the space charge density does not exceed 2%, since distribution of the charge is virtually unaffected.

As the probe is moved closer to the emitter-electrode, the volt-ampere characteristic of a spherical probe becomes distorted by the considerable increase of the volume charge in the sphere neighborhood, by the increase of the probe capacitance, and, also, by the growth of charge $Q_e$. The first two factors increase the slope of the volt-ampere characteristic because of the third which shift the point of intersection of characteristics with the axis $O\varphi_s$ to the left.

Consequently, the error of determination of local electrical parameters by an electric probe close to the source of charged particles in the main associated with the considerable perturbation of the electric charge distribution. The upper bound of the error introduced in the measurement of $q$ is defined by (2.4).

With diminishing degree of saturation of the stream by the electric charge, the error of measurement decreases. The upper diagram in Fig.4 shows the dependence of the current absorbed by the sphere on parameter $\beta$ for $x_0 = 0.25$; the dash line shows that dependence for a perfect gas, and curves 1, 2, and 3 relate to spheres of radii $a = 0.05$, 0.025, and 0.0125, respectively.

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